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DECOUPLING CONTROL OF SYSTEMS WITH UNCERTAIN PARAMETERS DEFINED OVER A DISCRETE RANGE

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DECOUPLING CONTROL OF SYSTEMS WITH UNCERTAIN PARAMETERS DEFINED OVER A DISCRETE RANGE

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Abstract

The problem of decoupling of systems with parameter uncertainty is treated. The possible parameter values are defined over a discrete range. The conditions for decoupling are given as well as for disturbance isolation.

I. INTRODUCTION

The problem of decoupling of systems with uncertainty is considered. The uncertainty is assumed to be contained in the system matrix which is parameterized by a parameter vector which is uncertain except that it is defined over a discrete range. The conditions for decoupling are developed for this type of system. The disturbance isolation problem is treated in Appendix B. This problem may correspond for a solution to a decoupled flight control application where the vehicle must be decoupled over a finite set of flight conditions (1).

The decoupling problem with uncertainty is treated in a different manner in references (3-4). In (3) the data sensitivity problem is treated where decoupling is assured for a class of perturbations in order to obtain what is known as output invariance decoupling but called data sensitivity in this reference. The results yield strong solutions. Therefore, the results herein will assure decoupling outside the allowable set of reference (3) but are limited to a discrete range of possible parameter values.

2. PROBLEM STATEMENT

Consider the linear time-invariant system

$$\dot{\underline{x}}(t) = \underline{A}(\Theta)\underline{x}(t) + \underline{B}\ \underline{u}(t) \tag{1}$$

where $\underline{x}\epsilon R^n$ is the system state, $\underline{u}\epsilon R^q$ is the system control, $\underline{B}$ is a n x m control gain matrix, and $\underline{A}(\Theta)$ is an n x n dynamic matrix parameterized by a time-invariant and uncertain parameter vector $\Theta\epsilon R^p$. Let $\hat{k}$ denote the index set $\{1,2,...k\}$. It is assumed that Θ is defined over a discrete range, i.e., $\Theta\epsilon\{\Theta_1,\Theta_2,...\Theta_k\}$ where the Θ_i's are the possible parameter values. The output vector is given by the equation

$$\underline{y}(t) = \underline{H}\ \underline{x}(t) \tag{2}$$

where $\underline{y}\epsilon R^m$ is the output vector and $\underline{H}$ is a m x n time-invariant output matrix. The control $\underline{u}(t)$ will be chosen in a static feedback manner as

$$\underline{u}(t) = \underline{F}\ \underline{x}(t) + \underline{G}\ \underline{v}(t) \tag{3}$$

where $\underline{v} \epsilon R^m$ is the external input to the closed loop system. The matrices $\underline{F}$ and $\underline{G}$ of obvious dimension are to be chosen such that the i-th element of the external input vector, $\underline{v}$, will control the i-th element and only the i-th element of the output vector, $\underline{y}$, independent of the value of Θ where Θ is defined over the discrete range. Thus, the problem that must be addressed is that of characterization of the class of decoupling control laws that will decouple the system irrespective of the parameter values within the discrete range.

The problem heuristically stated above will be mathematically formulated in terms of the geometric theory and the conditions for decoupling will be developed. Furthermore, Appendix B gives the results for disturbance isolation irrespective of the parameter value in the discrete range.

3. DIAGONAL DECOUPLING

In this section the mathematical problem formulation of the decoupling problem of systems with parameter values defined over a discrete range will be developed. It is shown that the decoupling problem solution consists of finding certain subspaces that have given requirements imposed upon them. The synthesis of such subspaces is considered in the next section.

One may rewrite the output equation (2) in terms of its elements, i.e.,

$$\underline{y}_j = \underline{H}_j \underline{x}, \quad j = 1,2,\ldots m \tag{4}$$

where $\underline{H}_j$ is the j-th row of the output matrix H. Furthermore, one may write the control law (3) as

$$\underline{u} = \underline{F}\,\underline{x} + \sum_{j=1}^{m} \underline{G}_j\,\underline{v}_j \tag{5}$$

where $\underline{G}_j$ is the j-th column of $\underline{G}$. It may be shown that the output space controllable by each $\underline{v}_j$ for a fixed O_i is given as

$$R_{ij}^{OUT} = \underline{H}_j\{\underline{A}(O_i) + \underline{B}\ \underline{F}|\{\underline{B}\ \underline{G}_j\}\} \tag{6}$$

In order to control $\underline{y}_j$ with $\underline{v}_j$ one must have

$$R_{ij}^{OUT} = H_j \tag{7}$$

where H_j is the range of $\underline{H}_j$. Therefore, in order to assure that one may control $\underline{y}_j$ completely with $\underline{v}_j$ irrespective of the parameter value O_i, $i\epsilon\hat{k}$, one must have

$$R_{ij}^{OUT} = H_j, \quad \forall i\epsilon\hat{k} \tag{8}$$

This equation may be rewritten as

$$\underline{H}_j\{\underline{A}(O_i) + \underline{B}\ \underline{F}|\{\underline{B}\ \underline{G}_j\}\} = H_j, \quad \forall i\epsilon\hat{k}. \tag{9}$$

Now if one denotes the subspace $\{\underline{A}(O_i) + \underline{B}\ \underline{F}|\{\underline{B}\ \underline{G}_j\}\}$ as R_{ij}, it is clear that in order for the above condition to be met one must have $R_{1_j} = R_{2_j} = \ldots = R_{k_j}$.

Therefore, in order to control the output element $\underline{y}_j$ completely with $\underline{v}_j$ irrespective of the parameter value O_i, $i\epsilon\hat{k}$ the output space for each value of O_i must equal the range space of $\underline{H}_j$. Furthermore, it is clear that the output space for each value of O_i must be equal.

Let the null space of $\underline{H}_j$ be denoted as $\underline{N}_j$. Given $(\underline{A}(O_i), i\epsilon\hat{k})$, $\underline{B}$, and $\underline{H}_j$, j=1,2,...m, the controllability subspace of $(\underline{A}(O_i), i\epsilon\hat{k}; \underline{B}\ \underline{G}_j)$ is given as

$$R_j = \{\underline{A}(O_i) + \underline{B}\ \underline{F}|\{\underline{B}\ \underline{G}_j\}\}, \quad \forall i\epsilon\hat{k}. \tag{10}$$

From reference (2) it is clear that one may rewrite this as

$$R_j = \{\underline{A}(O_i) + \underline{B}\ \underline{F}|B \cap R_j\}, \quad \forall i\epsilon k. \tag{11}$$

In order to assure noninteraction between the $\underline{v}_j$'s and the $\underline{y}_\ell$'s, $\ell \neq j$, one must have that

$$R_j \subset \bigcap_{\ell \neq j} N_\ell. \tag{12}$$

This will assure that the states controllable by each $\underline{v}_j$ being in the null space of $\underline{H}_\ell$, $\ell \neq j$. Furthermore, the condition

$$R_j + N_j = R^n, \quad j = 1,2,\ldots,m \tag{13}$$

must be satisfied in order to assure that one may control $\underline{y}_j$ completely with $\underline{v}_j$.

Thus, in summary the conditions that must be met in order to decouple the uncertain system are that

$$R_j = \{\underline{A}(O_i) + \underline{B}\ \underline{F} | B \cap R_j\}, \quad \forall i \in \hat{k}, \quad j = 1,2,\ldots,m$$

$$R_j \subset \bigcap_{\ell \neq j} N_\ell, \quad j = 1,2,\ldots,m$$

$$R_j + N_j = R^n, \quad j = 1,2,\ldots m. \tag{14}$$

Hence, given $(\underline{A}(O_i),\ i \in \hat{k})$, $\underline{B}$, and $N_1, N_2, \ldots, N_m$ the problem is that of determining the subspaces R_j, $j = 1,2,\ldots,m$ that satisfies the above conditions. These conditions are similar to those found in reference (2). However, the conditions are much more stringent in the requirements placed upon each of the possible subspaces R_{ij}.

This completes the mathematical problem formulation. The construction of the necessary subspaces will be developed in the next section.

4. CHARACTERIZATION OF THE CONTROLLABILITY SUBSPACES

In this section the necessary lemmas and theorems will be developed in order to characterize the required controllability subspaces. The bound on the maximum number of uncertain parameters within the discrete range is found.

Definition 1: The subspace ν is said to be invariant with respect to $(\underline{C}(O_i), i\varepsilon\hat{k})$ if $\underline{C}(O_i)\nu \subset \nu$, $\forall i\varepsilon\hat{k}$.

Lemma 1: Let $\nu \subset R^n$ and $\hat{k}$ be the index set $\hat{k} = \{1,2,\ldots,k\}$ where k is the number of matrices. Let $\hat{n} = \dim \nu$. There exists an $m \times n$ matrix F such that $\{\underline{A}(O_i) + \underline{B}\,\underline{F}\}\ \nu \subset \nu\ \forall i\varepsilon\hat{k}$ if and only if $\underline{A}(O_i)\ \ \subset \nu + B$, $\forall i\varepsilon\hat{k}$ and $[\underline{A}(O_\ell) - \underline{A}(O_j)]\ \nu \subset \nu$ for all ℓ, $j\varepsilon\hat{k}$.

Proof: Sufficiency. Let $\{v_1, v_2, \ldots, v_k\}$ be a set of basis vectors for ν. Assume that $\underline{A}(O_i)\ \nu \subset B + \nu$ for all $i\varepsilon\hat{k}$. Now, one has for some $u_j\varepsilon R^m$ and $w_j\varepsilon\nu$ and some $i\varepsilon\hat{k}$

$$\underline{A}(O_i)\underline{v}_j = \underline{B}\ \underline{u}_j + \underline{w}_j,\quad \forall i\varepsilon\hat{k}$$

and that

$$[\underline{A}(O_\ell) - \underline{A}(O_i)]\underline{v}_j = w_{j\ell m} \ \varepsilon\ \nu.$$

Now, choose F such that

$$\underline{F}\ \underline{v}_1 = -\underline{u}_1$$

$$\underline{F}\ \underline{v}_2 = -\underline{u}_2$$

$$\vdots$$

$$\underline{F}\ \underline{v}_{\hat{n}} = -\underline{u}_{\hat{n}}.$$

One has that

$$\underline{A}(O_i)\underline{v}_j = -\underline{B}\,\underline{F}\,\underline{v}_j + \underline{w}_j$$

or

$$[\underline{A}(O_i) + \underline{B}\,\underline{F}]\underline{v}_j = \underline{w}_j.$$

Furthermore, one has that

$$\underline{A}(O_\ell)\underline{v}_j = \underline{A}(O_i)\underline{v}_j - \underline{w}_{ij\ell}$$

or

$$\underline{A}(O_\ell)\underline{v}_j = -\underline{B}\,\underline{F}\,\underline{v}_j + \underline{w}_j - \underline{w}_{ij\ell}.$$

This may be rewritten as

$$[\underline{A}(O_\ell) + \underline{B}\,\underline{F}]\underline{v}_j = \underline{w}_j - \underline{w}_{ij\ell} \in \nu .$$

Therefore, this choice of F and the conditions on the matrices preserves the results.

Necessity. Assume that

$$[\underline{A}(O_i) + \underline{B}\,\underline{F}]\nu \subset \nu, \quad \forall i\varepsilon\hat{k}$$

Then for $v_j \varepsilon \nu$ there exists a $w_{ij} \varepsilon \nu$ such that

$$[\underline{A}(O_i) + \underline{B}\,\underline{F}]\underline{v}_j = \underline{w}_{ij}, \quad \forall i\varepsilon\hat{k}.$$

This implies that

$$\underline{A}(O_i)\underline{v}_j = \underline{w}_{ij} - \underline{B}\,\underline{F}\,\underline{v}_j, \quad \forall i\varepsilon\hat{k}$$

or

$$\underline{A}(O_i)\nu \subset \nu + B, \quad \forall i.$$

Now, one has that

$$[\underline{A}(O_i) + \underline{B}\ \underline{F}]\underline{v}_j = \underline{w}_{ij}$$

and

$$[\underline{A}(O_\ell) + \underline{B}\ \underline{F}]\underline{v}_j = \underline{w}_{\ell j}$$

Thus, one has that

$$[\underline{A}(O_i) - \underline{A}(O_\ell)]\underline{v}_j = \underline{w}_{ij} - \underline{w}_{ij} \in \nu$$

or

$$[\underline{A}(O_i) - \underline{A}(O_\ell)]\ \nu \subset \nu.$$

This lemma yields the necessary and sufficient conditions for the existence of a feedback matrix $\underline{F}$ that will assure that the subspace ν is invariant with respect to $(\underline{A}(O_i) + \underline{B}\ \underline{F},\ i\epsilon\hat{k})$.

Lemma 2: Let F be the class of matrices $\underline{F} \ni [\underline{A}(O\ ) + \underline{B}\ \underline{F}]R \subset R$, $\forall i\epsilon\hat{k}$. Let $\tilde{R} \subset R$. Now $\forall\ \underline{F} \epsilon F$

$$R \cap B + [\underline{A}(O_i) + \underline{B}\ \underline{F}]\tilde{R} = R \cap [\underline{A}(O_i)\tilde{R} + B],\quad \forall i\epsilon\hat{k}. \tag{15}$$

Proof: Let $\underline{F} \epsilon F$. Then $\{\underline{A}(O_i) + \underline{B}\ \underline{F}\}\tilde{R} \subset \tilde{R} \subset R$, $\forall i\epsilon\hat{k}$, and since $\underline{B}\ \underline{F}\ \tilde{R} \subset B$, then $\underline{A}(O_i)\tilde{R} + B = [\underline{A}(O_i) + \underline{B}\ \underline{F}]\tilde{R} + B$, $\forall i\epsilon\hat{k}$. Thus,

$$R \cap [\underline{A}(O_i)\tilde{R} + B] = R \cap \{[\underline{A}(O_i) + \underline{B}\ \underline{F}]\tilde{R} + B\},\quad \forall i\epsilon k.$$

One may use the distributive rule for subspaces

$$[L \cap (N+M \cap L)] = L \cap M + L \cap N,$$

which yields

$$R \cap [\underline{A}(O_i)\tilde{R}+B] = R \cap B + R \cap [\underline{A}(O_i)+\underline{B}\underline{F}]\tilde{R},\ \forall i\epsilon\hat{k}.$$

However, $\tilde{R} \subset R$ and $[\underline{A}(O_i)+\underline{BF}]\tilde{R} \subset \tilde{R}$. Thus,

$$R \cap [\underline{A}(O_i)\tilde{R}+B] = R \cap B + [\underline{A}(O_i) + \underline{BF}]\tilde{R}, \ \forall i \varepsilon \hat{k}.$$

Lemma 3: If $\underline{F} \varepsilon F$, then

$$\sum_{j=1}^{\ell} [\underline{A}(O_i)+\underline{BF}]^{j-1} B \cap R = R_i^{(\ell)}, \ i\varepsilon\hat{k} \ \ell = 1,2,\ldots,m \tag{16}$$

where

$$R_i^{(\ell)} = R \cap [\underline{A}(O_i)R_i^{(\ell-1)}+B], \ \forall i\varepsilon\hat{k}$$
$$R_i^{(0)} = 0. \tag{17}$$

Proof: $\ell = 1$

$$[\underline{A}(O_i)+\underline{BF}]^0 \ B \cap R \ = R \cap [\underline{A}(O_i)(0) \ B]$$
$$B \cap R \ = R \cap B, \ \forall i\varepsilon\hat{k}.$$

Assume it is true for $\ell = \rho - 1$. Then

$$\sum_{j=1}^{\rho} [\underline{A}(O_i)+\underline{BF}]^{j-1} B \cap R = B \cap R + \sum_{j=1}^{\rho} [\underline{A}(O_i)+\underline{BF}]^{j-1} B \cap R$$
$$= B \cap R + [\underline{A}(O_i)+\underline{BF}] \sum_{j=1}^{\rho-1} [\underline{A}(O_i)+\underline{BF}]^{j-} \ B \cap R$$
$$= B \cap R + [\underline{A}(O_i)+\underline{BF}] \ R_i^{(\rho-1)}, \ \forall i\varepsilon\hat{k}.$$

By use of Lemma 2 one obtains

$$\sum_{j=1}^{\rho} [\underline{A}(O_i)+\underline{BF}]^{j-1} B \cap R = R \cap [\underline{A}(O_i)R_i^{(\rho-1)} +B] \ \ \forall i\varepsilon\hat{k}.$$

As the conditions for decoupling require that the subspaces R_j, $j = 1,2,\ldots,m$ be found that satisfy the conditions given in equation (14), the method of synthesizing these subspaces must be developed. In particular given $(\underline{A}(O_i), \ i\varepsilon\hat{k})$, $\underline{B}$, and R one must find the conditions for the existence of $\underline{F} \varepsilon F$ such that

$$R = \{\underline{A}(O_i) + \underline{B}\,\underline{F}|B \cap R\}, \quad \forall i \epsilon \hat{k}. \tag{18}$$

If such an $\underline{F}$ exists then R is called a controllability subspace of $(\underline{A}(O_i),\ i\epsilon\hat{k};\ \underline{B})$.

Theorem 1: Given $(\underline{A}(O_i),\ i\epsilon\hat{k})$, $\underline{B}$, and $R \subset R^n$. R is a controllability subspace of $(\underline{A}(O_i),\ i\epsilon\hat{k};\ \underline{B})$ if and only if

$$\underline{A}(O_i)R \subset B+R, \quad \forall i\epsilon\hat{k}, \tag{19}$$

$$[\underline{A}(O_\ell) - \underline{A}(O_j)]R \subset R,\ \forall \ell,\ j\epsilon\hat{k},$$

$$R = R_i,\ \forall i\epsilon\hat{k} \tag{20}$$

where $\hat{R}_i$, $i\epsilon\hat{k}$, are the minimal subspaces such that

$$\hat{R}_i = R \cap [\underline{A}(O_i)\hat{R}_i + B],\ \forall i\epsilon\hat{k}. \tag{21}$$

Furthermore, $\hat{R}_i = R_i^{(\rho)}$, $\forall i\epsilon\hat{k}$ where $\rho = \dim R$ and

$$\begin{aligned} R_i^{(0)} &= 0 \\ R_i^{(\rho)} &= R \cap [\underline{A}(O_i)R_i^{(\rho-1)} + B],\ \forall i\epsilon\hat{k}. \end{aligned} \tag{22}$$

Proof: Assume R is a controllability subspace. Then

$$R = \{\underline{A}(O_i) + \underline{B}\,\underline{F}|B \cap R\},\ \forall i\epsilon\hat{k}.$$

Now $\underline{F} \epsilon F$ implies

$$[\underline{A}(O_i) + \underline{B}\,\underline{F}]R \subset R,\ \forall i\epsilon\hat{k}.$$

By Lemma 1

$$\underline{A}(O_i)R \subset R + B,\ \forall i\epsilon\hat{k}$$

and

$$[\underline{A}(O_\ell) - \underline{A}(O_j)]R \subset R,\ \forall \ell,\ j\epsilon\hat{k}$$

Furthermore,

$$R = \sum_{j=1}^{n} [\underline{A}(O_i)+\underline{B}\underline{F}]^{j-1} \mathcal{B} \cap R, \ \forall i \epsilon \hat{k}$$

$$= R_i^{(n)} = R_i^{(\rho)}$$

by Lemma 3.

Assume

$$A(O_i)R \subset \mathcal{B} + R$$

$$R = \hat{R}_i.$$

Then since

$$\hat{R}_i = R \cap [\underline{A}(O_i)\hat{R}_i + \mathcal{B}], \ \forall i \epsilon \hat{k}$$

$$= \sum_{j=1}^{n} [\underline{A}(O_i)+\underline{B}\underline{F}]^{j-1} \mathcal{B} \cap R, \ \forall i \epsilon \hat{k}$$

$$= \{\underline{A}(O_i) + \underline{B}\underline{F} | \mathcal{B} \cap R\} \ \ \forall i \epsilon \hat{k},$$

and $\underline{F} \epsilon F$. To show that the sequence has a minimal solution $R^{(\rho)}$ one may proceed by induction to show $R^{(\ell)} \subset \hat{R}$, $\ell = 1,2,\ldots$ for every solution $\hat{R}$ and that the sequence $R^{(\ell)}$ is monotone nondecreasing. Hence, there is a $\mu \le \rho \ni R^{(j)} = R^{(\mu)}$ for $j \ge \mu$ in particular $R^{(\rho)}$ satisfies the sequence.

In order to calculate the maximal controllability subspace, $\bar{R}$, contained in a given subspace L, let $\bar{v}$ be the maximal subspace of L which is $[\underline{A}(O_i)+\underline{B}\underline{F}]$ invariant for all $i \epsilon \hat{k}$ for some $\underline{F}$ and let $F(\bar{v})$ be the class of $\underline{F}$ such that $[\underline{A}(O_i)+\underline{B}\underline{F}]\bar{v} \subset \bar{v}$, $\forall i \epsilon \hat{k}$. Now, in order to find this subspace one may apply the following theorem.

Theorem 2: If $\underline{F} \epsilon F(\bar{v})$ the subspace

$$\bar{R} = \{\underline{A}(O_i)+\underline{B}\underline{F} | \mathcal{B} \cap \bar{v}\}, \ \forall i \epsilon \hat{k} \qquad (23)$$

is the maximal controllability subspace.

Proof: $\bar{R}$ is obviously a controllability subspace of $(\underline{A}(O_i), i\varepsilon\hat{k}; \underline{B})$. From the sequence in equation (22) one may see that this subspace is independent of $\underline{F} \varepsilon F(\bar{\nu})$ and, therefore, is uniquely defined. Now, let

$$\hat{R} = \{\underline{A}(O_i)+\underline{BF}|B \cap R\}, \ \forall i\varepsilon\hat{k}, \quad R \subset L.$$

Then, $\hat{R}$ is $[\underline{A}(O_i)+\underline{BF}]$ invariant $\forall i\varepsilon\hat{k}$ and since $\bar{\nu}$ is maximal, then $\hat{R} \subset \bar{\nu}$. Let $\bar{\nu} = \hat{R} \oplus \hat{\nu}$. Then there exists an $\underline{F}$ such that

$$[\underline{A}(O_i)+\underline{BF}]\hat{\nu} \subset \bar{\nu}, \ \forall i\varepsilon\hat{k}$$

$$\underline{Fx} = \underline{\hat{F}x}, \ x\varepsilon\hat{R}.$$

Then $\underline{F} \varepsilon F(\bar{\nu})$ and

$$\hat{R} = \{\underline{A}(O_i)+\underline{BF}|B \cap \hat{R}\} \subset \{\underline{A}(O_i)+\underline{BF}|B \cap \bar{\nu}\} = \bar{R}.$$

The next section gives the conditions for the existence of a solution to the problem in equation (14).

5. EXISTENCE OF A SOLUTION

This section gives the conditions that must be satisfied to yield a solution to the decoupling problem given in equation (14).

Theorem 3: If the dim $B = m$, then equation (14) has a solution if and only if

$$\bar{R}_j + N_j = R^n$$

and

$$B = \sum_{j=1}^{m} B \cap \bar{R}_j$$

where

$$\bar{R}_j = \{\underline{A}(O_i)+\underline{BF}|B \cap \bar{\nu}_j\}, \ \forall i\varepsilon\hat{k}.$$

Furthermore, if $\underline{F}$, R_1, R_2, ..., R_m is any solution to (14), then

$$R_j = \bar{R}_j, \ j = 1,2,\ldots,m.$$

Proof: The proof follows in a similar manner to Theorem 7.1 in reference (2) with appropriate modifications.

6. CONCLUSIONS

The problem formulation for decoupling of systems with uncertain parameters defined over a discrete range is given. It is shown that the maximum number of parameters that can be in the discrete ran e is limited to the dimension of the maximal invariant subspace as defined prior to Theorem 2. The results are given to ensure the decoupling irrespective of the parameter value $\Theta \varepsilon R^p$ within the discrete range. The problem of disturbance isolation of systems with uncertain parameters within a discrete range is treated in Appendix B.

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APPENDIX A

CONSTRUCTION LEMMA

APPENDIX A

Lemma A.1: Let $\underline{x}_i \varepsilon R^n$, $\underline{u}_i \varepsilon R^m$, $i = 1,2,\ldots,n$. There exists an $m \times n$ matrix $\underline{F}$ such that $\underline{F}\,\underline{x}_i = \underline{u}_i$, $\forall i \varepsilon \{\hat{k}, k+1, \ldots, n\}$, where $\hat{k}$ is the index set $\hat{k} = \{1,2,\ldots,k\}$ and $k \leq n$, if and only if $N(\underline{X}) = N(\underline{U})$ where $\underline{F}$ and $\underline{U}$ are matrices with column vectors $\underline{x}_i$ and $\underline{u}_i$, respectively. If the $\underline{x}_i$'s are linearly independent $\underline{F}$ always exists.

Proof: One may assume an $\underline{F}$ exists such that $\underline{F}\,\underline{x}_i = \underline{u}_i$, $\forall i \varepsilon \hat{k}$ and that the $\underline{x}_i$'s are linearly independent. Thus, the rank of $\underline{x}$ is n and

$$\underline{X}\,\underline{y} = 0$$

has no nontrivial solutions and, therefore, $N(\underline{X}) = 0$. It follows that $N(\underline{X}) \subset N(\underline{U})$ since $N(\underline{U})$ contains at least the null vector and $N(\underline{X})$ contains at most the null vector. Now, the following equation may be formed

$$\underline{F}\,\underline{X} = \underline{U}$$

where

$$\underline{F}\,\underline{X} = \underline{F}\{\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_{k-1}, \underline{x}_k, \underline{x}_{k+1}, \ldots, \underline{x}_n\}$$

and

$$\underline{U} = \{\underline{u}_1, \underline{u}_2, \ldots, \underline{u}_{k-1}, \underline{u}_k, \underline{u}_{k+1}, \ldots, \underline{u}_n\}$$

Now,

$$\underline{X}^T\,\underline{F}^T = \underline{U}^T.$$

This equation has a solution for $\underline{C}$ if each column of $\underline{U}^T$ lies in the range space of $\underline{X}^T$, i.e., let $\underline{a}_i$ denote the i-th column of $\underline{U}^T$. Then

$$\underline{a}_i \in R\,(\underline{X}^T)$$

for each $i = 1,2,\ldots,n$. Let $\sum_{i=1}^{n} \gamma_i\,\underline{a}_i$ denote the span of the $\underline{a}_i$'s denoted

by $[\underline{A}]$. Then

$$[\underline{A}] \subset R(\underline{X}^T)$$

and

$$R(\underline{U}^T) \subset R(\underline{X}^T).$$

From reference (5)

$$N(\underline{A}^T) = R(\underline{A})^{\perp}.$$

Let $\underline{A} = \underline{B}^T$, then

$$N(\underline{B}) = R(\underline{B}^T)^{\perp}$$

which implies

$$N(\underline{B})^{\perp} = R(\underline{B}^T).$$

Therefore,

$$N(\underline{U})^{\perp} \subset R(\underline{X}^T) = N(\underline{X})^{\perp}$$

and

$$N(\underline{U})^{\perp} \subset N(\underline{X})^{\perp}$$

which implies

$$N(\underline{U}) \subset N(\underline{X}).$$

APPENDIX B

DISTURBANCE ISOLATION OF SYSTEMS WITH UNCERTAIN PARAMETERS

APPENDIX B

Consider the system

$$\dot{\underline{x}} = \underline{A}(\Theta_i)\underline{x} + \underline{B}\,\underline{u} + \underline{D}\,\underline{\xi} \tag{B.1}$$

with

$$\underline{u} = \underline{F}\,\underline{x} + \underline{v} \tag{B.2}$$

and

$$\underline{y} = \underline{H}\,\underline{x}. \tag{B.3}$$

The parameter vector Θ is uncertain but defined over a discrete range as in Section 2. The output $\underline{y}$ will be unaffected by $\underline{\xi}$ irrespective of the parameter Θ_i, $\forall i\varepsilon\hat{k}$ if and only if

$$\{\underline{A}(\Theta_i) + \underline{B}\,\underline{F}|\mathcal{D}\} \subset N(\underline{H}), \ \forall i\varepsilon\hat{k}. \tag{B.4}$$

Theorem B.1: There exists an F such that $\{\underline{A}(\Theta_i) + \underline{B}\,\underline{F}|\mathcal{D}\} \subset N(\underline{H})$, $\forall i\varepsilon\hat{k}$ if and only if $\mathcal{D} \subset \nu$ where ν is the maximal subspace such that

$$\nu \subset N(H) \cap \underline{A}^{-1}(\Theta_1)(\mathcal{B}+\nu) \cap \underline{A}^{-1}(\Theta_2)(\mathcal{B}+\nu) \cap \ldots \cap \underline{A}^{-1}(\Theta_k)(\mathcal{B}+\nu) \tag{B.5}$$

and

$$[\underline{A}(\Theta_\ell) - \underline{A}(\Theta_j)]\nu \subset \nu, \ \forall \ell,\ j\varepsilon\hat{k}.$$

Furthermore, ν is given by $\nu = \nu^{(r)}$ where

$$\nu^{(\ell)} = \nu^{(\ell-1)} \cap \underline{A}^{-1}(\Theta_1)(\mathcal{B}+\nu^{(\ell-1)}) \cap \underline{A}^{-1}(\Theta_2)(\mathcal{B}+\nu^{(\ell-1)}) \cap \ldots$$
$$\ldots \cap \underline{A}^{-1}(\Theta_k)(\mathcal{B}+\nu^{(\ell-1)}). \tag{B.6}$$

and

$$r = \dim N(\underline{H})$$

Proof: Now,

$$\nu \subset N(\underline{H}) \cap \underline{A}^{-1}(\Theta_1)(\mathcal{B}+\nu) \cap \underline{A}^{-1}(\Theta_2)(\mathcal{B}+\nu) \cap$$
$$\ldots \cap \underline{A}^{-1}(\Theta_k)(\mathcal{B}+\nu)$$

implies that $\nu \subset N(\underline{H})$ and $\nu \subset \underline{A}^{-1}(O_i)(B+\nu)$, $\forall i\epsilon\hat{k}$. Thus

$$\underline{A}(O_i)\nu \subset B+\nu, \ \forall i\epsilon\hat{k}.$$

By Lemma 1 there exists an $\underline{F}$ such that

$$\{\underline{A}(O_i) + \underline{B}\ \underline{F}\}\ \nu \subset \nu, \ \forall i\epsilon\hat{k}.$$

Now, $\mathcal{D} \subset \nu$ implies that

$$\{\underline{A}(O_i) + \underline{B}\ \underline{F}|\mathcal{D}\} \subset \{\underline{A}(O_i) + \underline{B}\ \underline{F}|\nu\}, \ \forall i\epsilon\hat{k}.$$

But

$$\{\underline{A}(O_i) + \underline{B}\ \underline{F}|\nu\} = \nu + (\underline{A}(O_i) + \underline{B}\ \underline{F})\ \nu + \ldots + (\underline{A}(O_i) + \underline{B}\ \underline{F})^{n-1}\nu$$

and

$$(\underline{A}(O_i) + \underline{B}\ \underline{F})\ \nu \subset \nu, \ \forall i\epsilon\hat{k}$$

imply that

$$\underline{A}(O_i) + \underline{B}\ \underline{F}|\nu\} = \nu, \ \forall i\epsilon\hat{k}.$$

where

$$\nu \subset N(\underline{H})$$

Thus,

$$\{\underline{A}(O_i) + \underline{B}\ \underline{F}|\mathcal{D}\} \subset \{\underline{A}(O_i) + \underline{B}\ \underline{F}|\nu\} = \nu \subset N(\underline{H}), \ \forall i\epsilon\hat{k}.$$

One may assume there exists an $\underline{F}$ such that

$$\{\underline{A}(O_i) + \underline{B}\ \underline{F}|\mathcal{D}\} \subset N(\underline{H}), \ \forall i\epsilon\hat{k}.$$

Let

$$\{\underline{A}(O_i) + \underline{B}\ \underline{F}|\mathcal{D}\} = \mathcal{W}_i \subset N(\underline{H}), \ \forall i\epsilon\hat{k}.$$

Now,

$$\{\underline{A}(O_i) + \underline{B}\ \underline{F}\}\ \mathcal{W}_i = \{\underline{A}(O_i) + \underline{B}\ \underline{F}\}\{\mathcal{D} + (\underline{A}(O_i) + \underline{B}\ \underline{F})\mathcal{D} + \ldots + (\underline{A}(O_i) + \underline{B}\ \underline{F})^{n-1}\mathcal{D}\} \subset (\quad, \ \forall i\epsilon\hat{k}.$$

Since W_i is a cyclic subspace it is invariant with respect to $\underline{A}(O_i)+\underline{B}\,\underline{F}$. Furthermore, from Lemma A.1 one has

$$\underline{A}(O_i)\, W_i \subset B + W_i,\ \forall i\varepsilon\hat{k}$$

and

$$[\underline{A}(O_\ell) - A(O_j)]\, W_j \subset W_j,\ \forall \ell,\ j\varepsilon\hat{k}.$$

Thus,

$$\left.\begin{array}{l} \underline{A}(O_i)W_i \subset B + W_i \\ W_i \subset N(\underline{H}) \end{array}\right\} \forall i\varepsilon\ .$$

An upper bound for each W_i is $N(\underline{H})$.

In order to show the existence of a maximal subspace ν such that

$$\underline{A}(O_i)\nu \subset B+\nu,\ \forall i\varepsilon\hat{k}$$

$$\nu \subset N(\underline{H})$$

one may show the existence of the maximal subspaces

$$\underline{A}(O_i)\, \overline{W}_i \subset B + \overline{W}_i,\ \forall i\varepsilon\hat{k}$$

$$\overline{W}_i \subset N(\underline{H}).$$

The required maximal subspace ν is then given by

$$\nu = \bigcap_{i\varepsilon\hat{k}} \overline{W}_i.$$

It follows from (2) that the maximal subspaces $\overline{W}_i$ exist. Thus, the required subspace is given by equation (B.6). It may be easily proven th

$$\underline{A}(O_i) \bigcap_{i\varepsilon\hat{k}} \overline{W}_i \subset B + \bigcap_{i\varepsilon\hat{k}} \overline{W}_i,$$

$$\bigcap_{i\varepsilon\hat{k}} \overline{W}_i \subset N(\underline{H}).$$

In order to show that ν is a subspace one may note that ν is nonempty since each $\bar{W}_i$ contains the zero vector. Furthermore, since $\bar{W}_i$'s, $\forall i \epsilon \hat{k}$ a e subspaces as can easily be shown, then from the well known theorem th t the intersection of subspaces is a subspace it follows that ν is a sub ace.

Now, since $\mathcal{V} \subset W_i \subset \bar{W}_i$, $\forall i \epsilon \hat{k}$, then

$$\mathcal{V} \subset \nu.$$

In order to compute the maximal subspace in $N(\underline{H})$ we may use the following algorithm. We need to compute the maximal subspace in $N(\underline{H})$ which satisfies the requirements that

$$\nu \subset N(\underline{H})$$

$$\underline{A}(O_i)\nu \subset \mathcal{B} + \nu, \ \forall i \epsilon \hat{k}.$$

Let $\nu^{(0)} = N(\underline{H})$, then $\nu \subset \nu^{(0)}$. Let

$$\nu^{(j+1)} = \nu^{(j)} \cap \underline{A}^{-1}(O_1)(\mathcal{B}+\nu^{(j)}) \cap \ldots$$

$$\cap \underline{A}^{-1}(O_k)(\mathcal{B}+\nu^{(j)})$$

Assume $\nu \subset \nu^{(j)}$. Then $\nu \subset \nu^{(j+1)}$. Thus, $\nu \subset \nu^{(j)}$, $\forall j$.

Furthermore the sequence $\nu^{(0)}, \nu^{(1)}, \ldots \nu^{(\ell)}$ is monotone-decreasing. Since $N(\underline{H})$ is finite dimension, there exists an integer which is less than dim $N(\underline{H}) \ni \nu^{(j)} = \nu^{(\ell)}$ for all $j \geq \ell$. Since $\nu \subset \nu^{(\ell)}$ and $\nu^{(\ell)}$ satisfies

$$\nu^{(\ell)} \subset N(\underline{H})$$

$$\underline{A}(O_i)\nu^{(\ell)} \subset \mathcal{B} + \nu^{(\ell)}, \ \forall i \epsilon \hat{k},$$

then $\nu = \nu^{(\ell)}$.